MATHEMATICS

1. Answer any four of the following: (4x7½=30)

(a) Let $\{e_1, e_2, e_3, e_4\}$ be a basis for a vector space V over R. Prove that $\{e_1 - e_2, e_2 - e_3, e_3 - e_4, e_4 - e_1\}$ is also a basis of V.

(b) Let g: R – R be defined by : g(t) = 0 if t is irrational or 0

$$= \frac{1}{N} \text{ if } t = \frac{m}{n}$$

Where m and n are integers, t is non-zero and heighest common factor of m and n is l.
Prove that g is continuous at all irrational t and discontinuous at all rational non-zero t.

(c) Find the equation of the plane passing through the line:

$$\frac{x-1}{4} = \frac{y-2}{6} = \frac{z-1}{3}$$

And the point (4. 3. 7).

(d) Let ABCD be a square. Suppose forces represented in magnitude and direction by AB, 2BC, 2CD, DA and DB are acting at a point 0. Prove that they are at equilibrium.

(e) A truck is moving along a level road at the rate of 40 km/hr. In what direction a bullet must Be fired from it with a velocity of 200 m/sec so that its resultant motion is perpendicular to the truck?

(f)A random variable x follows Poisson distribution such that P(x = 1) is equal to P (x=2). Find P (x>3).

PART I

2.(i) Let T : $R^2 - R^2$ be the linear tranformation defined by :

T(x,y) = (2x + 3y, y + 3x)

Find the matrix of T with respect to the basis { (1, 1), (1, -1)} (10)

(ii) Find the matrix P such that P'AP is diagonal where P' denotes the transpose of P and A is the matrix: (12)

1	2	-1
2	0	3
-1	3	1

(iii) Let ƛ and μ be distinct eigen values of a Hermitian matrix H. Suppose x and y are eigen vectors corresponding to ƛ and μ respectively. Prove the x and y are mutually orthogonal. (8)

3.(i) Prove that if f : [0,1] – R is continuous on [0, 1] except at finitely many points, then f is Riemann integrable. (10)

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(ii) Find the volume of the torus generated by revolving the circle:

$X^2 + y^2 = 4$

About the line x = 3

(iii) Determine the points function:

$X^3 + y =$

Has a maximum or minimum. (5)

(iv) Find the radious of curvature of the curve:

$X^{2/3} + y^{(2/3)} = a^{(2/3)}$

At the point ($a \cos^3\theta$, $a \sin^3\theta$) (5)

4.(i) Solve the differential equation: (8)

$Y \sin 2x\, dx – (1 + y^2 + \cos x)\, dy = 0$

(ii) Solve: (12)

$(D^2 + a^2)\, y = \sin ax.$

(iii) Solve:

$(2x^2y – 3y^4)dx + (3x^2 + 2xy^3)dy = 0$ (10)

PART II

5.(i) Find curl grad F, where $F= X^2y + 2xyz + z^2$ (8)

(ii) If r and a are two vectors, prove that curl (rxa) = - 2a (7)

(iii) State Gauss's divergence theorem and use it to evaluate

$X^2\, dx\, dz + y^2\, dz\, dx + 2z(xy – x – y)\, dx\, dy$

Where S is the surface of the cube

$0 \le x \le 1$, $0 \le y \le 1$ and $0 \le z \le 1$. (15)

6.(i) Prove that a continuous real valued function defined on [3, 8] is uniformly continuous on [3, 8].(10)

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